The Ninth Grade Math Competition Class

Radical Expressions and Rationalizing Denominators Problems

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1. Find $\sqrt{9+\sqrt{56}} - \sqrt{9-\sqrt{56}}$. $= \sqrt{7}+\sqrt{2} - (\sqrt{7}-\sqrt{2}) = 2\sqrt{2}$

$\sqrt{9+\sqrt{56}} = \sqrt{a}+\sqrt{b} = \sqrt{7}+\sqrt{2}$

$9+\sqrt{56} = a+b+2\sqrt{ab}$

$9 = a+b$ $\quad \sqrt{56} = 2\sqrt{ab} = \sqrt{4ab}$

$7+2$ $\quad 14 = ab$

$7 \cdot 2$

$\sqrt{9-\sqrt{56}} = \sqrt{7}-\sqrt{2}$

2. Rationalize the denominator of $\frac{1}{2-\sqrt[3]{2}}$.

$$\frac{1}{2-\sqrt[3]{2}}\,\frac{(2^2+2\sqrt[3]{2}+\sqrt[3]{4})}{(2^2+2\sqrt[3]{2}+\sqrt[3]{4})}=\frac{4+2\sqrt[3]{2}+\sqrt[3]{4}}{8-2}$$

$$(a-b)(a^2+ab+b^2)=a^3-b^3$$

$$=\frac{4+2\sqrt[3]{2}+\sqrt[3]{4}}{6}$$

3. Rationalize the following denominator $\frac{8}{\sqrt{15}-\sqrt{7}}$.

$$\frac{8}{\sqrt{15}-\sqrt{7}} \cdot \frac{(\sqrt{15}+\sqrt{7})}{(\sqrt{15}+\sqrt{7})} = \frac{8\sqrt{15}+8\sqrt{7}}{\underbrace{15-7}_{8}}$$

$$= \sqrt{15}+\sqrt{7}$$

4. In how many real values of x is $\sqrt{120 - \sqrt{x}}$ an integer?

$$120 - \sqrt{x} \geq 0$$

$$120 - \sqrt{x} = k^2 \qquad k^2 = \left\{ \begin{array}{l} 10^2 \\ \vdots \\ 1^2 \\ 0^2 \end{array} \right.$$

11 solutions

$$120 - \sqrt{x} = 0$$

$$\sqrt{x} = 120$$

$$x = 120^2$$

5. Let $a^2 = \frac{4}{11}$, $b^2 = \frac{(2+\sqrt{5})^2}{11}$, where a is a negative real number and b is a positive real number. If $(a+b)^3$ can be expressed in the simplified form $\frac{x\sqrt{y}}{z}$, where x, y, z are positive integers. Find $x + y + z$.

$$a^2 = \frac{4}{11} \qquad b^2 = \frac{(2+\sqrt{5})^2}{11}$$

$$a = -\frac{2}{\sqrt{11}} \qquad b = \frac{2+\sqrt{5}}{\sqrt{11}}$$

$$(a+b)^3 = \left(-\frac{2}{\sqrt{11}} + \frac{2+\sqrt{5}}{\sqrt{11}}\right)^3 = \left(\frac{\sqrt{5}}{\sqrt{11}}\right)^3 = \frac{5\sqrt{5}}{11\sqrt{11}}$$

$$\frac{5\sqrt{5}}{11\sqrt{11}} \frac{\sqrt{11}}{\sqrt{11}} = \frac{5\sqrt{55}}{121}$$

$$5 + 55 + 121 = 181$$

6. Rationalize the denominator of $\frac{1}{\sqrt[3]{2}+\sqrt[3]{16}}$.

$\sqrt[3]{2^3\cdot 2} = 2\sqrt[3]{2}$

$\frac{1}{\sqrt[3]{2}+\sqrt[3]{16}}$

$\frac{1}{\sqrt[3]{2}+2\sqrt[3]{2}} = \frac{1}{3\sqrt[3]{2}} \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{6}$

$(a+b)(a^2-ab+b^2) = a^3+b^3$

7. What is the product of the real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$.

$a = x^2 + 18x + 45$

$a - 15 = 2\sqrt{a}$

$a^2 - 30a + 225 = 4a^2$

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8. Determine the rational number $\frac{a}{b}$ in lowest terms that equal to

$$\frac{1}{\sqrt{2}+2}+\frac{1}{2\sqrt{3}+3\sqrt{2}}+\frac{1}{3\sqrt{4}+4\sqrt{3}}+\cdots+\frac{1}{(2013^2-1)\sqrt{2013^2}+2013^2\sqrt{2013^2-1}}$$

$x=2$ $\quad x=3$ $\quad x=4$ $\quad x=2013^2$

$$\frac{1}{(x-1)\sqrt{x}+x\sqrt{x-1}}\cdot\frac{((x-1)\sqrt{x}-x\sqrt{x-1})}{((x-1)\sqrt{x}-x\sqrt{x-1})}$$

$$=\frac{(x-1)\sqrt{x}-x\sqrt{x-1}}{x^3-2x^2+x-x^3+x^2}$$

$$=\frac{(x-1)\sqrt{x}-x\sqrt{x-1}}{-x(x-1)}$$

$$=\frac{\sqrt{x}}{-x}-\frac{\sqrt{x-1}}{-(x-1)}=-\frac{\sqrt{x}}{x}+\sqrt{\ }$$

$$-\cancel{\frac{\sqrt{2}}{2}}+\frac{\sqrt{1}}{1}-\cancel{\frac{\sqrt{3}}{3}}+\cancel{\frac{\sqrt{2}}{2}}-\cdots-\frac{2013}{2013^2}+\cancel{\frac{\sqrt{2013^2-1}}{(2013^2-1)}}$$

$$=\frac{1}{1}-\frac{1}{2013}=\frac{2012}{2013}$$