The Ninth Grade Math Competition Class
Modular Arithmetic
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1. The remainders when two natural numbers are divided by 12 are 5 and 9. (a) Find the remainder when their product is divided by 12. (b) Find the reminder when their product is divided by 4.

2. Is $21^{100} - 12^{100}$ a multiple of 11?

3. Find the remainder when $24^{50} - 15^{50}$ is divided by 13.

4. Find the tens and units digits of 7^{2006}.

5. Find the remainder when $1^2 + 2^2 + 3^2 + \cdots + 99^2$ is divided by 13.

6. Find the remainder when $9^{42} - 5^{42}$ is divided by 7.

7. Find the remainder when 7^{255} is divided by 11.

$3^{2020} \mod 10$ $\quad$ $7^{255} \mod 11$

$3^1 = 3$

$3^2 = 9$

$\vdots$

Fermat's Little Theorem

$a^{p-1} \equiv 1 \pmod p$ $\quad$ a is any #, p is a prime

$a=7,\ p=11$

$7^{10} \equiv 1 \pmod{11}$

$7^{255} \equiv 7^{25 \cdot 10} \cdot 7^5 \equiv (7^{10})^{25} 7^5 \equiv 7^5 \pmod{11}$

8. Find the last two digits of 99^{2005}.

$99^{2005} \mod 100 = -1 \pmod{100}$

$99 \equiv -1 \pmod{100}$

$(-1)^{2005} \equiv -1 \pmod{100}$

9. A natural number n, has a unit digit of A when expressed in base 12. Find the remainder when n^2 is divided by 6.

$$49 \bmod 10 = 9$$

$$n_{12} \bmod 12 = 10$$

$$n \equiv 10 \pmod{12}$$

$$n^2 \equiv 100 \pmod{12}$$

$$n^2 \equiv 4 \pmod{12}$$

$$n^2 - 4 = 12k$$

$$n^2 = 12k + 4$$

$$\boxed{n^2 \equiv 4 \pmod{6}}$$