The Ninth Grade Math Competition Class
Linear Equations and Proportions
Anthony Wang

1. Solve for t such that $\frac{\sqrt{3+\sqrt[3]{t}}}{\sqrt{3-\sqrt[3]{t}}} = 3$.

$$\frac{3+\sqrt[3]{t}}{3-\sqrt[3]{t}} = 9$$

$$3+\underbrace{\sqrt[3]{t}}_{x} = 9\left(3-\underbrace{\sqrt[3]{t}}_{x}\right)$$

$$3+x = 27-9x$$

$$10x = 24$$

$$x = \frac{12}{5}$$

$x = \sqrt[3]{t}$

$$\frac{12}{5} = \sqrt[3]{t}$$

$$t = \left(\frac{12}{5}\right)^3 = \frac{1728}{125}$$

$$= 13.824$$

2. Find x and y values such that

$$\frac{3x-4y}{xy} = -8$$
$$\frac{2x+7y}{xy} = 43.$$

$3x - 4y = -8(xy)$

$3 \cdot \frac{1}{y} \quad 4 \cdot \frac{1}{x}$

$\frac{1}{y} = a$

$\frac{1}{x} = b$

$\frac{3x}{xy} - \frac{4y}{xy} = -8$

$\frac{3}{y} - \frac{4}{x} = -8$

$3a - 4b = -8$

$6a - 8b = -16$

$\frac{2}{y} + \frac{7}{x} = 43$

$2 \cdot \frac{1}{y} \quad 7 \cdot \frac{1}{x}$

$6a + 21b = 129$

$2a + 7b = 43$

$29b = 145$

$\frac{1}{y} = 4 \quad y = \frac{1}{4}$

$b = 5$

$\frac{1}{x} = 5 \quad x = \frac{1}{5}$

$a = 4$

3. Assume that $X_1, X_2, \ldots, X_7$ are real numbers such that

$$\begin{aligned} X_1 + 4X_2 + 9X_3 + 16X_4 + 25X_5 + 36X_6 + 49X_7 &= 1 \quad (1) \\ 4X_1 + 9X_2 + 16X_3 + 25X_4 + 36X_5 + 49X_6 + 64X_7 &= 12 \quad (2) \\ 9X_1 + 16X_2 + 25X_3 + 36X_4 + 49X_5 + 64X_6 + 81X_7 &= 123 \quad (3) \end{aligned}$$

Find the value of $16X_1 + 25X_2 + 36X_3 + 49X_4 + 64X_5 + 81X_6 + 100X_7$. $= A$ (4)

$3X_1 + 5X_2 + 7X_3 + \ldots 15X_7 = 11$ (2)-(1)

$5X_1 + 7X_2 + \ldots + 17X_7 = 111$ (3)-(2)

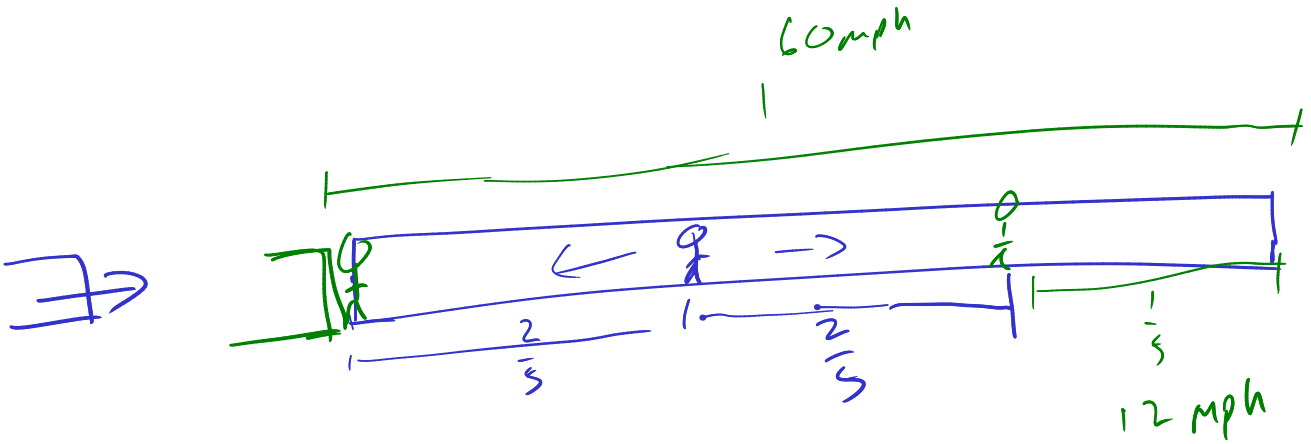
$2X_1 + 2X_2 + 2X_3 + \ldots 2X_7 = 100$

$7X_1 + 9X_2 + \ldots + 19X_7 = A - 123$ (4)-(3)

$111 + 100 = A - 123$

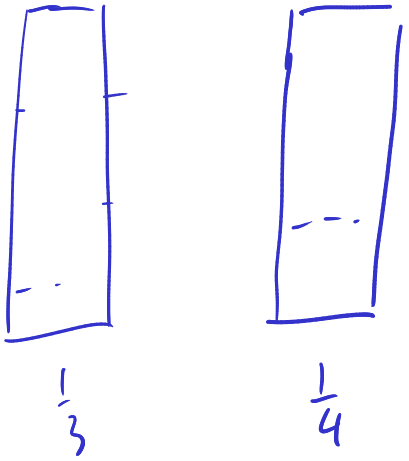
$A = \boxed{334}$

4. A man is running through a train tunnel, and when he is $\frac{2}{5}$ of the way through, he hears a train that's approaching the tunnel from behind him at a speed of 60mph. Whether he runs ahead or runs back, he will reach an end of the tunnel at the same time as the train reaches the end. At what rate is he running?



5. Two candles of the same length are made of different materials so that one burns out completely at a uniform rate in 3 hours and the other in 4 hours. At what time in the afternoon, should the candles be lit so that at 4pm one stub is twice the length of the other?

$t = 1:36$ pm



$$2\left(1 - \underbrace{\tfrac{1}{3}(4-t)}_{\text{burned}}\right) = 1 - \tfrac{1}{4}(4-t)$$

$$2 - \tfrac{2}{3}(4-t) = 1 - 1 + \tfrac{1}{4}t$$

$$2 - \tfrac{8}{3} + \tfrac{2}{3}t = \tfrac{1}{4}t$$

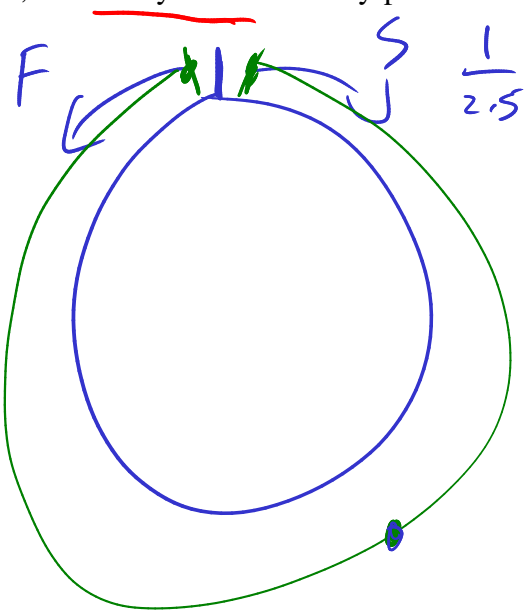
$$\left(\tfrac{2}{3} - \tfrac{1}{4}\right)t = \tfrac{8}{3} - 2$$

$$\tfrac{8-3}{12}t = \tfrac{2}{3}$$

$$t = \tfrac{2}{3} \cdot \tfrac{\overset{4}{\cancel{12}}}{5} = \tfrac{8}{5}$$

$$= 1\tfrac{3}{5}$$

6. Superman and Flash are running around the world in opposite directions. Superman can go around the world in 2.5 hours, and Flash can do the same in 1.5 hours. Assuming they start at the same time and same place, how many times will they pass each other going in opposite directions in 24-hour period?



$\frac{1}{1.5} = \frac{2}{3}$

$\frac{1}{2.5} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$

$\frac{1}{\frac{2}{5}+\frac{2}{3}} = \frac{1}{\frac{6+10}{15}} = \frac{15}{16}$ hr

$\frac{24}{\frac{15}{16}} = 25.6$

25

7. For three distinct positive numbers x, y, z,

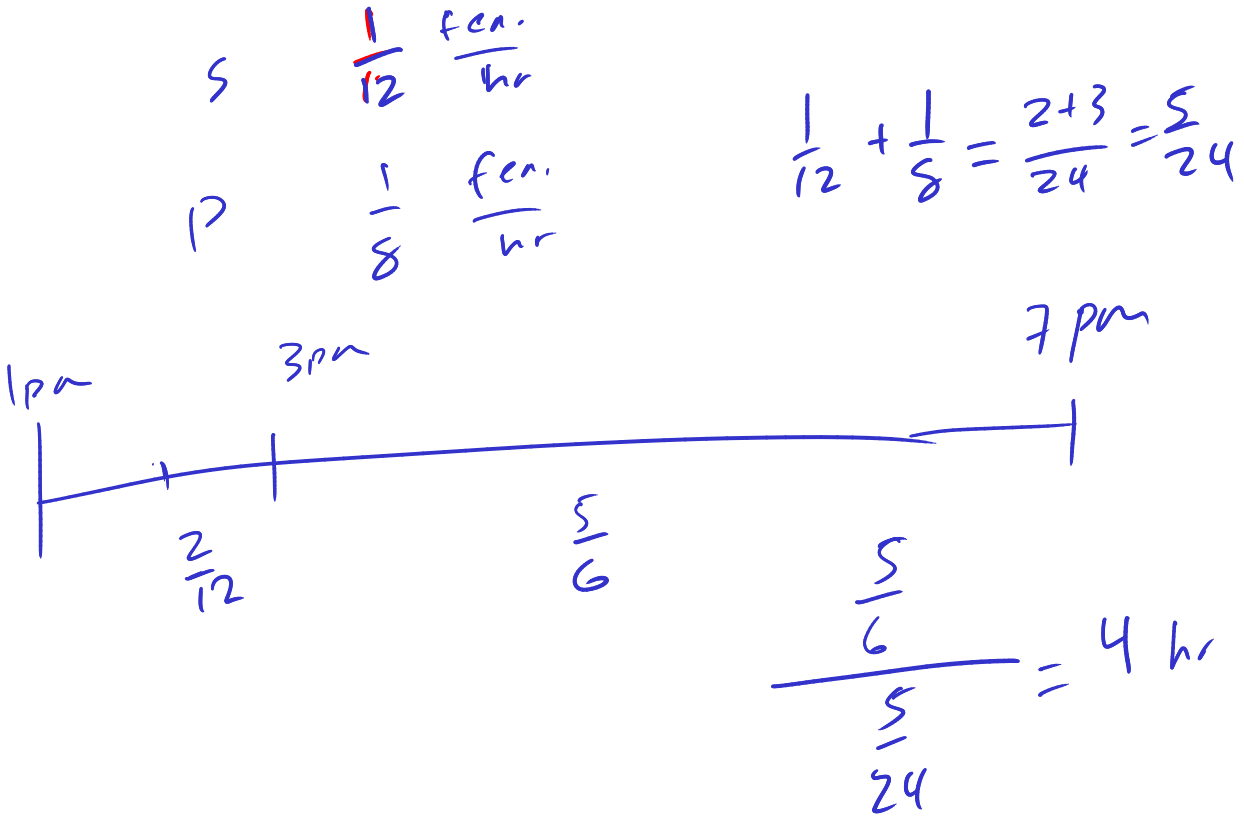
$$\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y} = k = 2$$

find $\frac{x}{y}$.

$$\frac{y+x+y+x}{x-z+z+y} = k$$

$$\frac{\overbrace{2(x+y)}}{x+y} = k = 2$$

8. Peter and Sam are painting a fence. Sam could paint the whole fence alone in 12 hours, Peter could paint the whole fence in 8 hours. Sam starts painting at 1p.m. and Peter joins him at 3p.m. At what time do they finish?



9. Solve for x

$$\frac{1}{x-2} = \frac{2x-1}{x^2-x-2}$$

$x = 2 \rightarrow \frac{1}{0}$ X

No solutions