The Ninth Grade Math Competition Class

Exponents

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1. Find $5^{-3}5^{5}5^{1}$.

$5^{-3+5+1} = 5^{3}$

2. Find $\frac{3^4 3^{-2}}{3^5 3^{-1}}$. $= \dfrac{3^2}{3^4} = 3^{2-4} = 3^{-2} = \dfrac{1}{9}$

$3^4 3^{-2} = 3^{4-2} = 3^2$

$3^5 3^{-1} = 3^{5-1} = 3^4$

3. Find 4^{x+1} if 2^x is 9.

$2^x = 9$

$4^{x+1} = 4^x \cdot 4^1 = 4 \cdot 4^x$

$= 4 \cdot (2^2)^x = 4 \cdot 2^{2x} = 4 \cdot (2^x)^2$

$\Rightarrow 4^{x+1} = 4 \cdot 9^2 = 324$

4. If $8^x = 27$, what is 4^{2x-3}.

$8^x = 27$ $\qquad (2^3)^x = \boxed{27 = 2^{3x} = (2^x)^3}$

$4^{2x-3} = \dfrac{4^{2x}}{4^3} = \dfrac{(2^2)^{2x}}{4^3} = \dfrac{2^{4x}}{4^3}$

$= \dfrac{(2^x)^4}{4^3} = \dfrac{3^4}{4^3} = \dfrac{81}{64}$ $\qquad 27 = (2^x)^3$

$3 = 2^x$

5. Find all values of x such that $25^{-2} = \frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} 25^{\frac{17}{x}}}$.

$$(5^2)^{-2} = \frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} (5^2)^{\frac{17}{x}}}$$

$$5^{-4} = \frac{5^{\frac{48}{x}}}{5^{\frac{26}{x}} 5^{\frac{34}{x}}} = 5^{\frac{48}{x} - \frac{26}{x} - \frac{34}{x}}$$

$$5^{-4} = 5^{-\frac{12}{x}}$$

$$-4 = -\frac{12}{x} \Rightarrow x = 3$$

6. Simplify the expression $81^{-(2^{-2})}$.

$$81^{-\frac{1}{4}} = \frac{1}{81^{\frac{1}{4}}} = \frac{1}{3} = 3^{-1}$$

7. Find x if $2^{16^x} = 16^{2^x}$.

$$2^{16^x} = (2^4)^{2^x}$$

$$2^{16^x} = 2^{4 \cdot 2^x}$$

$$16^x = 4 \cdot 2^x$$

$$2^{4x} = 2^2 \cdot 2^x$$

$$2^{4x} = 2^{2+x}$$

$$4x = 2 + x$$

$$x = \frac{2}{3}$$

8. Solve for n: $\sqrt{1+\sqrt{2+\sqrt{n}}}=2$.

9. Find, with a rational common denominator, the sum

$$(\frac{1}{2})^{-\frac{1}{2}} + (\frac{3}{2})^{-\frac{3}{2}} + (\frac{5}{2})^{-\frac{5}{2}}$$

10. What is the difference between $x^2 = 9$ and $x = \sqrt{9}$?

11. Suppose that $y = \frac{3}{4}x$ and $x^y = y^x$, the quantity $x + y$ can be expressed as a rational number $\frac{r}{s}$, where r and s are relatively prime positive integers. Find $r + s$.

$$y = \frac{3}{4}x \qquad y = \frac{4^3}{3^3}$$

$$x^y = y^x$$

$$x^{\frac{3}{4}x} = \left(\frac{3}{4}x\right)^x$$

$$x^{\frac{3}{4}x} = \left(\frac{3}{4}\right)^x x^x$$

$$\frac{x^{\frac{3}{4}x}}{x^x} = \left(\frac{3}{4}\right)^x$$

$$x^{-\frac{1}{4}x} = \left(\frac{3}{4}\right)^x$$

$$x^{-\frac{1}{4}} = \frac{3}{4} \qquad x = \left(\frac{3}{4}\right)^{-4} = \frac{4^4}{3^4}$$

$$\frac{4^3}{3^3} + \frac{4^4}{3^4} = \frac{\overset{r}{448}}{\underset{s}{81}}$$

529

12. The formula $N = 8 * 10^8 * x^{-\frac{3}{2}}$ gives, for a certain group, the number of individuals whose income exceeds x dollars. What is the smallest possible value of the lowest income of the wealthiest 800 individuals?

$$800 = 8 \cdot 10^8 x^{-\frac{3}{2}}$$

$$\left(10^{-6}\right)^{-\frac{2}{3}} = \left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}}$$

$$x = 10^4 = 10000$$

13. Solve for x in the equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1}$.

$\Rightarrow x = \frac{2}{111}$

$2^{111x} = y = 4 = 2^2$

$111x = 2$

$$\frac{2^{333x}}{2^2} + 2^2 \cdot 2^{111x} = 2^1 \cdot 2^{222x}$$

$$\frac{\left(2^{111x}\right)^3}{4} + 4 \cdot 2^{111x} = 2 \cdot \left(2^{111x}\right)^2$$

$$\frac{y^3}{4} + 4 \cdot y = 2 \cdot y^2$$

$$\frac{y^2}{4} + 4 = 2y$$

$$y^2 - 8y + 16 = 0$$

$$(y-4)^2 = 0 \quad \Rightarrow y = 4$$