The Ninth Grade Math Competition Class
Unit Digit
Anthony Wang

1. What is the units digit of $(133^{13})^3$?

$(133^{13})^3 = 133^{13 \cdot 3} = 133^{39}$

$\to 3^{39}$ $\to 7$

2. Find the units digit of n given that $m.n = 21^6$ and m has a units digit of 7.

$m \cdot n = 21^6 \rightarrow 1^6 = 1$

$m \cdot n \rightarrow 1$

$7 \cdot n \rightarrow 1$

$7 \cdot 1 \rightarrow 7$ ✗

$7 \cdot 3 \rightarrow 1$ ✓

$7 \cdot 5 \rightarrow 5$ ✗

$7 \cdot 7 \rightarrow 9$ ✗

$7 \cdot 9 \rightarrow 3$ ✗

3. (a:) Find the units digit of the sum

$$1! + 2! + 3! + \cdots + 2006!$$

$1+2+6+4 \to 3$

(b:) Find the units digit of the above sum when it is expressed in base 7.

$1! = 1$

$2! = 2$

$3! = 6$

$4! \to 4$

$\cdot 5$

$5! \to 0$

$\cdot 6$

$6! \to 6$

$7! \to 0$

$8! \to 0$

$\vdots$

$1! = 1$

$2! = 2$

$3! = 6$

$4! \to 24_{10} = 33_7$

$2 \cdot 10 + 4 = 3 \cdot 7 + 3$

$5! \to 15_{10} = 21$

$6! \to 6_7$

$7! \to 0$

$8! \to 0$

$9! \to 0$

$1+2+6+3+1+6 = 19_{10}$

$= 25_7$

4. A positive two-digit integer is divisible by n and its units digit is n. What is the greatest possible value of n?

$0 \leq n \leq 9$

$9 \mid 99$ ✓

5. Find the units digit of $3^{2016} - 2^{2016}$.

$11 - 6 = 5$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 \to 6$

$2^5 \to 2$

$2^6 \to 4$

$2^7 \to 8$

$2^8 \to 6$

$2^9 \to 2$

6. The cube of the 3-digit natural number $A7B$ is 108531333. What is $A + B$? $4+7=11$

$A7B^3 = 108,531,333$

$4 \quad 7 \quad 7$

$\approx 108 \cdot 10^6$

$1^3 = 1$

$A77 \approx \sqrt[3]{108} \cdot 100$

$4.8 \cdot 100$

$3^3 \to 7$

$A77 \approx 480$

$5^3 \to 5$

4

$7^3 \to 3$

$9^3 \to 9$

7. How many of the positive divisors of 6^{2006} have a units digit of 6?

8. **(a)** Convert 1599 to base 16.

(b) Find all possible units digits of perfect fourth powers when written in base 16.

(c) Determine all non-negative integral soulations $(n_1, n_2, \ldots, n_{14})$ if any, of the Diophantine equation.

$$n_1^4 + n_2^4 + \cdots + n_{14}^4 = 1599.$$

(A Diophantine equation is an equation in which only integer solutions are allowed.)

(a) $1599 = \underline{1600} - 1$

$16 \overline{)100}$

64_{16}

$640_{16} - 1 = 63F_{16}$

(b) $0^4 = 0$

$1^4 = 1$

$2^4 = 16 \to 0_{16}$

$3^4 = 81 \to 1_{16}$

$(2k)^4 = 16k^4 \to 0_{16}$

$(2k+1)^4 = \underline{16k^4} + \underline{32k^3} + 24k^2 + 8k + 1$

$8(3k^2+k)+1$

8. (a) Convert 1599 to base 16.

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(c) Determine all non-negative integral soulations $(n_1, n_2, \ldots, n_{14})$ if any, of the Diophantine equation.

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No solutions

$n_1^4 + n_2^4 + \ldots n_{14}^4 = 63F_{16}$

0 0 0 . . 0

15

1 1 1 1

$[0, 14]$