

The Ninth Grade Math Competition Class
Decimals
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1. Convert repeating decimal $0.\overline{3123}$ to fraction.

$$x = 0.\overline{3123} = 0.3123\overline{3123} \\ 10000x = 3123.\overline{3123}$$

$$9999x = 3123$$

$$x = \frac{3123}{9999}$$

$$\begin{array}{r} 347 \\ \hline 1111 \end{array}$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$3! = 3 \cdot 2 \cdot 1$$

$$2! = 2 \cdot 1$$

2. Compute $\frac{4+3!}{3!+2!}$. Express your answer as a decimal to the nearest hundredth.

$$\frac{4 \cdot 3 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 + 2 \cdot 1} = \frac{24 + 6}{6 + 2} = \frac{30}{8}$$

$$= \frac{15}{4} = 3 \frac{3}{4} = 3.75$$

3. What is the 4037th digit following the decimal point in the expansion of $\frac{1}{111}$?

$$\frac{1}{111} = \frac{9}{999} = \overline{.009}$$

1 2 3

4. Evaluate the infinite geometric series

$$x = \frac{7^0}{100} + \frac{7^1}{100^2} + \frac{7^2}{100^3} + \dots$$

as a fraction and find the first 6 digits in its decimal expansion.

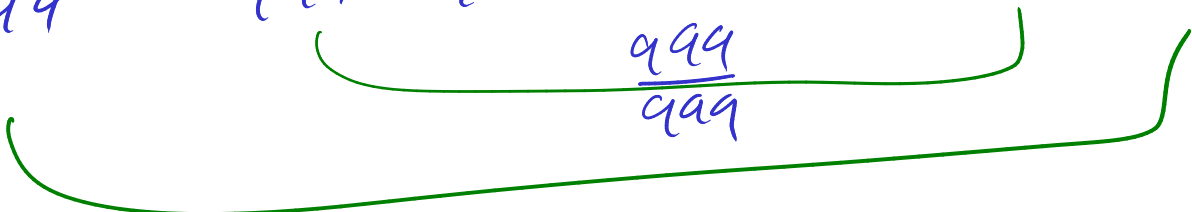
$$x = \frac{7^1}{100} + \frac{7^2}{100^2} + \frac{7^3}{100^3} + \dots$$

$$\frac{93}{100} x = \frac{7^0}{100} \Rightarrow x = \frac{1}{93}$$

.01
 .0007
 .000049
 .00000343
.010752

5. Let S be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$, $= \frac{abc}{999}$ where a, b, c are distinct digits. Find the sum of the elements of S .

$$\frac{012}{999} + \frac{013}{999} + \frac{014}{999} + \dots + \frac{986}{999} + \frac{987}{999}$$



 $\frac{999}{999}$

$$10 \cdot 9 \cdot 8 = 720 \text{ choices for } a, b, c$$

$$\frac{720}{2} = \textcircled{360}$$

6. The rational number r is the largest number less than 1 whose base-7 expansion consists of two distinct digits, i.e., $r = 0.\overline{AB}$. Written as a reduced fraction, $r = \frac{p}{q}$, find $p + q$.

$$x = .\overline{65}_7$$
$$100_7 x = 65.\overline{65}_7$$

$$66_7 x = 65_7$$

$$x = \frac{65_7}{66_7} = \frac{47}{48}$$

$$47 + 48 = 95$$

7. Express $0.72\overline{45}$ as a common fraction.

$$x = .72\overline{45} = .72\cancel{45}\cancel{45}\cancel{45}\cancel{45} \dots$$
$$100x = 72.\cancel{45}\cancel{45}\dots$$

$$99x = 71.73$$

$$x = \frac{71.73}{99} = \frac{7173}{9900}$$

8. Let p be a prime number other than 2 or 5. What is the maximum possible number of digits in the repeating block of digits in $\frac{1}{p}$?

$p-1$

$p \overline{) 1}$

. 1428571

$$\begin{array}{r}
 7 \overline{) 1.0} \\
 \underline{- 7} \\
 30 \\
 \underline{- 28} \\
 20 \\
 \underline{- 14} \\
 60 \\
 \underline{- 56} \\
 40 \\
 \underline{- 35} \\
 50 \\
 \underline{- 49} \\
 10 \\
 \underline{- 7} \\
 3
 \end{array}$$