The Ninth Grade Math Competition Class
Decimals
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1. Convert repeating decimal $0.\overline{3123}$ to fraction.

$x = 0.\overline{3123} = 0.312331233123\ldots$

$10000x = 3123.31233123\ldots$

$9999x = 3123$

$x = \frac{3123}{9999} = \frac{347}{1111}$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$
$$3! = 3 \cdot 2 \cdot 1$$
$$2! = 2 \cdot 1$$

2. Compute $\frac{4!+3!}{3!+2!}$. Express your answer as a decimal to the nearest hundredth.

$$\frac{4 \cdot 3 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 + 2 \cdot 1} = \frac{24+6}{6+2} = \frac{30}{8}$$

$$= \frac{15}{4} = 3\frac{3}{4} = 3.75$$

3. What is the 4037^{th} digit following the decimal point in the expansion of $\frac{1}{111}$?

$$\frac{1}{111} = \frac{9}{999} = .\overline{009}$$

1 2 3

4. Evaluate the infinite geometric series

$$x = \frac{7^0}{100} + \frac{7^1}{100^2} + \frac{7^2}{100^3} + \cdots$$

as a fraction and find the first 6 digits in its decimal expansion.

$$\frac{7}{100}x = \frac{7^1}{100^2} + \frac{7^2}{100^3} + \frac{7^3}{100^4}$$

$$\frac{93}{100}x = \frac{7^0}{100} \Rightarrow x = \frac{1}{93}$$

.01
.0007
.000049
.00000343
.010752

5. Let S be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$, $= \frac{abc}{999}$ where a, b, c are distinct digits. Find the sum of the elements of S.

$$\frac{012}{999} + \frac{013}{999} + \frac{014}{999} \cdots - - + \frac{986}{999} + \frac{987}{999}$$

$$\frac{999}{999}$$

$$\frac{999}{999}$$

$10 \cdot 9 \cdot 8 = 720$ choices for a, b, c

$$\frac{720}{2} = 360$$

6. The rational number r is the largest number less than 1 whose base-7 expansion consists of two distinct digits, i.e., $r = 0.\overline{AB}$. Written as a reduced fraction, $r = \frac{p}{q}$, find $p + q$.

$x = .\overline{65}_7$

$100_7 x = 65.\overline{65}_7$

$66_7 x = 65_7$

$x = \frac{65_7}{66_7} = \frac{47}{48}$

$47 + 48 = 95$

7. Express $0.72\overline{45}$ as a common fraction.

8. Let p be a prime number other than 2 or 5. What is the maximum possible number of digits in the repeating block of digits in $\frac{1}{p}$?