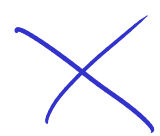


The Ninth Grade Math Competition Class
Divisibility Rules
Anthony Wang

1. What is the least number greater than 9000 that is divisible by 11?

9001 is this div. by 11? 

9002

$$9001 \equiv -9 + 0 - 0 + 1 \equiv -9 + 1 \equiv -8 \pmod{11}$$

$$9002 \equiv -9 + 0 - 0 + 2 \equiv -9 + 2 \equiv -7 \pmod{11}$$

$$9003 \equiv -9 + 0 - 0 + 3 \equiv -9 + 3 \equiv -6 \pmod{11}$$

$$\textcircled{9009} \equiv -9 + 0 - 0 + 9 \equiv -9 + 9 \equiv 0 \pmod{11}$$

$$900A \equiv -9 + 0 - 0 + A \equiv -9 + A \equiv 0 \pmod{11}$$

$$\textcircled{A=9}$$

2. Find A such that $3A6$ is a multiple of 9.

$$3 + A + 6 \equiv 0 \pmod{9}$$

$$A \equiv 0 \pmod{9}$$

$$A = 0, 9$$

3. Find the ordered pairs of digits (A, B) such that $67A7B$ is a multiple of 225.

$$225 = 3^2 \cdot 5^2$$

check div. by

25



$$B=5$$

$$67A75$$

$$6+7+A+7+5 \equiv 0 \pmod{9}$$

$$25+A \equiv 0 \pmod{9}$$

$$A=2, B=5$$

4. Find the value of the digit D if $47D4$ leaves a remainder of 2 when divided by 33.

47D4
↙

$$33 = 3 \cdot 11$$

$$-4 + 7 - D + 2 \equiv 5 - D \equiv 0 \pmod{11}$$

$$D = 5$$

5. A four-digit number uses each of the digits 1, 2, 3 and 4 exactly once. Find the probability that the number is a multiple of 4.

$$\frac{2 \cdot 3^2}{4!} = \frac{6}{24} = \frac{1}{4}$$

ABCD

12
13
41

$D = 2$

- | | |
|----|---|
| 12 | ✓ |
| 32 | ✓ |
| 42 | ✗ |

$D = 4$

- | | |
|----|---|
| 14 | ✗ |
| 24 | ✓ |
| 34 | ✗ |

6. Find the ordered pair of digits (M, N) such that $52MN5$ is a multiple of 1125.

$$1125 = 3^2 \cdot 5^3$$

$$52 \overset{MN}{1} 25 \equiv 15$$

X

$$52375 \equiv 22$$

X

$$52625 \equiv 20$$

X

$$52875 \equiv 27$$

✓

7. For all integer values of $n \geq 2$, k will divide $n^3 - n$. What is the greatest possible integer value of k ?

$$\begin{aligned}n^3 - n &= n(n^2 - 1) \\ &= n(n-1)(n+1)\end{aligned}$$

$$k \mid n(n-1)(n+1)$$

7, 8, 9
9, 10

$$k=2 \quad \checkmark$$

$$k=3 \quad \checkmark$$

$$k=4 \quad \times$$

$$k=5 \quad \times$$

$$k=6 \quad \checkmark$$

$$4 \nmid 2 \cdot 1 \cdot 3 = 6$$

$$4 \mid 9 \cdot 8 \cdot 10$$

8. The integer n is the smallest positive multiple of 15 such that every digit of n is either 0 or 8. Compute $\frac{n}{15}$.

$$15 = 3 \cdot 5$$

$$n = \underline{\underline{8880}}$$

$$\frac{n}{15} = \textcircled{592}$$