**The Ninth Grade Math Competition Class**
**Divisibility Rules**
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**1.** What is the least number greater than $9000$ that is divisible by $11$?

9001 is thus div. by 11?

9002

$9001 \equiv -9+0-0+1 \equiv -9+1 \equiv -8 \pmod{11}$

$9002 \equiv -9+0-0+2 \equiv -9+2 \equiv -7 \pmod{11}$

$9003 \equiv -9+0-0+3 \equiv -9+3 \equiv -6 \pmod{11}$

$9009 \equiv -9+0-0+9 \equiv -9+9 \equiv 0 \pmod{11}$

$900A \equiv -9+0-0+A \equiv -9+A \equiv 0 \pmod{11}$

$A = 9$

**2.** Find $A$ such that $3A6$ is a multiple of 9.

$$3 + A + 6 \equiv 0 \pmod 9$$

$$A \equiv 0 \pmod 9$$

$$A = 0, 9$$

**3.** Find the ordered pairs of digits $(A, B)$ such that $67A7\underline{B}$ is a multiple of 225.

$225 = 3^2 \cdot 5^2$

check div. by 25

$B = 5$

$67A75$

$6+7+A+7+5 \equiv 0 \pmod 9$

$25 + A \equiv 0 \pmod 9$

$A = 2, \; B = 5$

**4.** Find the value of the digit $D$ if $47D4$ leaves a remainder of $2$ when divided by $33$.

$47D2$

$33 = 3 \cdot 11$

$-4+7-D+2 \equiv 5-D \equiv 0 \pmod{11}$

$D = 5$

**5.** A four-digit number uses each of the digits 1, 2, 3 and 4 exactly once. Find the probability that the number is a multiple of 4.

$$\frac{2 \cdot 3}{4!} = \frac{6}{24} = \frac{1}{4}$$

$AB\underline{CD}$

12

~~13~~

~~41~~

$D = 2$

12 ✓

32 ✓

42 ✗

$D = 4$

14 ✗

24 ✓

34 ✗

**6.** Find the ordered pair of digits $(M, N)$ such that $52MN5$ is a multiple of 1125.

$1125 = 3^2 \cdot 5^3$

$$\begin{array}{lll} 52\overset{M}{1}\overset{N}{2}5 \equiv 15 & & \times \\ 52375 \equiv 22 & & \times \\ 52625 \equiv 20 & & \times \\ 52875 \equiv 27 & & \checkmark \end{array}$$

**7.** For all integer values of $n \geq 2$, $k$ will divide $n^3 - n$. What is the greatest possible integer value of $k$?

$$n^3 - n = n(n^2 - 1^2)$$
$$= n(n-1)(n+1)$$

$$k \mid n(n-1)(n+1)$$

7, 8, 9

8, 9, 10

$k=2$ ✓

$k=3$ ✓

$k=4$ ✗

$k=5$ ✗

$k=6$ ✓

$n=2$

$4 \nmid 2 \cdot 1 \cdot 3 = 6$

$4 \mid 9 \cdot 8 \cdot 10$

**8.** The integer $n$ is the smallest positive multiple of 15 such that every digit of $n$ is either 0 or 8. Compute $\frac{n}{15}$.

$15 = 3 \cdot 5$

$n = \_\,\underline{8}\,\underline{8}\,\underline{8}\,0$

$\frac{n}{15} = 592$