The Ninth Grade Math Competition Class

Triangles: area and the three centers

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1. In rectangle $ABCD$, $AB = 5, BC + 3$, F, G are on CD such that $DF = 1$ and $GC = 2$, AF and BG intersect at E. Find the area of $\triangle AEB$.

2. In the figure below, $ABCD$ has sides $AB = 6$, $CD = 8$, $BC = DA = 2$, $AB \parallel CD$, $PQ \parallel AD$, $PR \parallel BC$, $AP = PB$. Find $\frac{PX}{YR}$.

3. Rectangle $ABCD$, points F and G are on AB such that $AF = FG = GB$, E is midpoint of DC. AC intersects EF at H, and EG at J. The area of rectangle $ABCD$ is 70. Find the area of $\triangle AHF$.

4. $\triangle ABC$ has $AB = 9$, $BC = 12$, $CA = 15$, find the length of the median BD.

5. Given $AB = BC = 10$, $AC = 12$, find the circumradius and inradius of $\triangle ABC$.

6. AD bisects $\angle BAC$, I is the incenter of $\triangle ABC$, $AB = 7$, $BC = 8$, $AC = 11$, find $\frac{AI}{ID}$.

7. $\triangle ABC$, altitude AD intersects angle bisector BE at point X. If $\angle BAC = 117°$, $\angle ACB = 35°$, find $\angle DXE$.

8. Medians AX and BY of triangle ABC are perpendicular at point O, $AX = 12$, $BC = 10$, find AO, BY and median CE.

9. In $\triangle ABC$, $\angle C = 90°$, M is the mid point of BC, N is the mid point of AC, O is the mid point of MN. Perimeter of $\triangle ABC$ is 112, $ON = 12.5$, find area of $MNAB$.

10. The circle in the above has radius 1 and is circumscribed about equilateral triangle ABC. If X is mid point of AC, Y is on $\overset{\frown}{AC}$ such that $\angle YXA = 90°$, what is XY?

11. Point N is on hypotenuse BC of right triangle ABC such that $\angle CAN = 45°$, $AC = 8$, $AB = 6$, find AN.

12. For $\triangle ABC$, median AD and CE intersects at P, $PE = 1.5$, $PD = 2$, $DE = 2.5$, find the area of $AEDC$.

13. For $\triangle ABC$, $AB = 6, BC = 10, AC = 8$, D is the mid point of BC, what is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$.

14. A triangle with sides of 5, 12, 13 has both an inscribed and a circumscribed circle, what is the distance between the centers of those circles?

15. Find the area of $\triangle ABC$ with $AB = 13$, $AC = 14$, and $AB = 15$.

16. $\triangle XOY$ is a right triangle with $\angle XOY = 90°$, M, N are mid points of OX and OY, $XN = 19$, $YM = 22$, find XY.

17. Find the area of rhombus $ABCD$ given that the radii of the circles circumscribed around triangles ABD and ACD are 12.5 and 25, respectively.

18. As shown in the figure, triangle ABC is divided into six smaller triangles by lines drawn from the vertices through a common interior point. The areas of four of these triangles are as indicated. Find the area of triangle ABC.

19. In triangle ABC, $AB = 13$, $BC = 14$, $AC = 15$, and point G is the intersection of the medians. Points A', B', and C', are the images of A, B, and C, respectively, after a $180°$ rotation about G. What is the area of the union of the two regions enclosed by the triangles ABC and $A'B'C''$?

20. A point P is chosen in the interior of $\triangle ABC$ such that when lines are drawn through P parallel to the sides of $\triangle ABC$, the resulting smaller triangles t_1, t_2, and t_3 in the figure, have areas 4, 9, and 49, respectively. Find the area of $\triangle ABC$.