

Simplifying Radical Expressions

sqrt, cube roots
nth roots

$$\begin{aligned}\sqrt{18} + \sqrt{32} &= \\ &= \sqrt{3^2 \cdot 2} + \sqrt{2^5} \\ &= \sqrt{3^2} \cdot \sqrt{2} + \sqrt{2^4} \cdot \sqrt{2} \\ &= 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{9095625} &= \sqrt{3^3 \cdot 5^4 \cdot 7^2 \cdot 11} = \sqrt{3^2 \cdot 5^4 \cdot 7^2} \cdot \sqrt{33} \\ &= 3 \cdot 5^2 \cdot 7 \sqrt{33} \\ &= 525\sqrt{33}\end{aligned}$$

Ex: $\sqrt{3-2\sqrt{2}} = \sqrt{a-b}$
or $\sqrt{a+b}$

$$3-2\sqrt{2} = (\sqrt{a}-\sqrt{b})^2 = a+b-2\sqrt{ab}$$

$$\text{or } 3-2\sqrt{2} = (\sqrt{a}+\sqrt{b})^2 = a+b+2\sqrt{ab}$$

$$3 = a+b$$

$$2 = ab$$

Ex: Find all solutions to

Squared both sides $\left\{ \begin{aligned} \sqrt{x^2+9} &= 2x-3 \\ \sqrt{x^2+9} &= 2 \cdot 0 - 3 \\ x^2+9 &= (2x-3)^2 = 4x^2-12x+9 \end{aligned} \right.$

$$0 = 3x^2 - 12x = 3x(x-4)$$

$x=0$ $x=4$

"extraneous"

Rationalizing Denominator

$$\frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

Rational #: $\frac{p}{q}$ p, q integers

$$\frac{1}{\sqrt{7}-\sqrt{6}} \cdot \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$(a-b)(a+b) = a^2 - b^2$$

Ex: Find $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}$

$$\frac{1}{\sqrt{1}+\sqrt{2}} \cdot \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} = \frac{\sqrt{2}-\sqrt{1}}{2-1} = \sqrt{2}-\sqrt{1}$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \sqrt{3}-\sqrt{2}$$

$$\frac{1}{\sqrt{3}+\sqrt{4}} \cdot \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} = \sqrt{4}-\sqrt{3}$$

$$\cancel{\sqrt{2}-\sqrt{1}} + \cancel{\sqrt{3}-\sqrt{2}} + \cancel{\sqrt{4}-\sqrt{3}} + \dots + \cancel{\sqrt{100}-\sqrt{99}}$$

$0 - 1 = -1$ "telescoping"