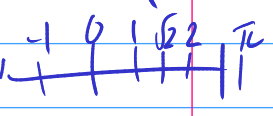
Complex Numbers

Integers: whole numbers: $-2, -1, 0, 1, 3 \ldots$

Rationals: fractions $\frac{p}{q}$ p, q are integers $q \neq 0$

Irrationals: $\pi, e, \sqrt{2}$

Real numbers



$\sqrt{2} = \frac{p}{q}$ ← Reduced form

$2 = \frac{p^2}{q^2}$

$2q^2 = p^2$ ⇒ p is even

$p = 2k$

$2q^2 = (2k)^2 = 4k^2$

$q^2 = 2k^2$ ← q is even

Imaginary: ai, a is real

$i = \sqrt{-1}$

~~$i^2 = -1$~~

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

i

Q: $a + b = 3$ $\quad ab = 16$

$b = \frac{16}{a}$

$a + \frac{16}{a} = 3$

$a^2 + 16 = 3a$ $\quad a^2 - 3a + 16 = 0$

$a = \frac{3 + \sqrt{55}i}{2}$

$b = \frac{3 - \sqrt{55}i}{2}$

$ab = \frac{(3+\sqrt{55}i)(3-\sqrt{55}i)}{4} = \frac{9 + 3\sqrt{55}i - 3\sqrt{55}i - \sqrt{55}^2 i^2}{4}$

$a = \frac{3 \pm \sqrt{9 - 4 \cdot 16}}{2} = \frac{3 \pm \sqrt{-55}}{2}$

$a = \frac{3 \pm \sqrt{55}\sqrt{-1}}{2} = \frac{3 \pm \sqrt{55}i}{2}$

complex: $a+bi$ a, b real

$\frac{9+55}{4} = \frac{64}{4} = 16$

$i \quad i^2 = -1 \quad i^3 = i^2 \cdot i = -i \quad i^4 = i^3 \cdot i = -i \cdot i = 1$

$i^5 = i^4 i = i \quad i^6 = i^5 i = -1 \quad i^7 = -i \quad i^8 = 1$

$i^{1007} = i^{1004+3} = i^{1004} \cdot i^3 = i^3 = -i$

$i^6 + i^{16} + i^{26} = -1 + (-1) = -1$

Adding complex #:

$3+4i + (-3+8i) = 0 + 12i = 12i$

Multiplying

$(3+4i)(-3+8i) = -9 + 18i - 12i + 4i \cdot 8i$

$4i \cdot 8i = -32$

$= -41 + 6i$

$z = 7+3i$

$w = 5-3\pi i$

$\bar{w} = 5+3\pi i$

$\overline{\bar{w}} = 5-3\pi i$

Complex conjugate $\bar{z} = 7-3i$

$z\bar{z} = (7+3i)(7-3i) = 49 + \cancel{7 \cdot 3i} - \cancel{7 \cdot 3i} - 9i^2$

$-9i^2 = +9$

$= 58$

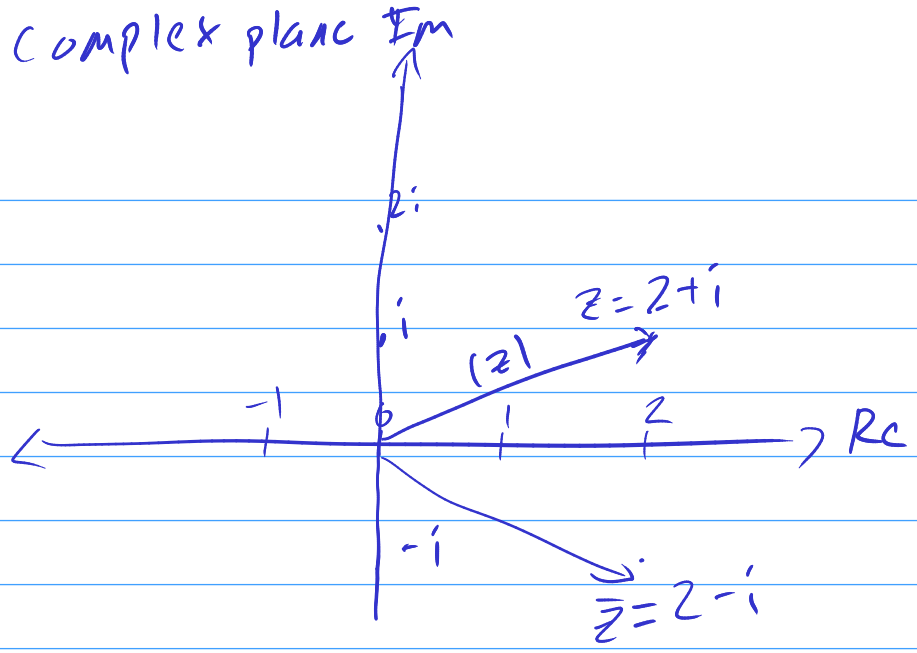
$z = a+bi$

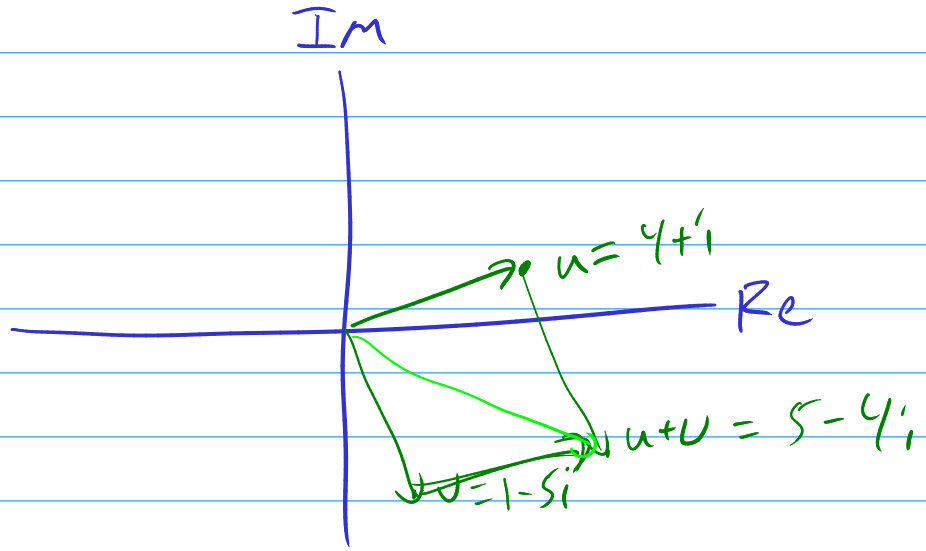
magnitude

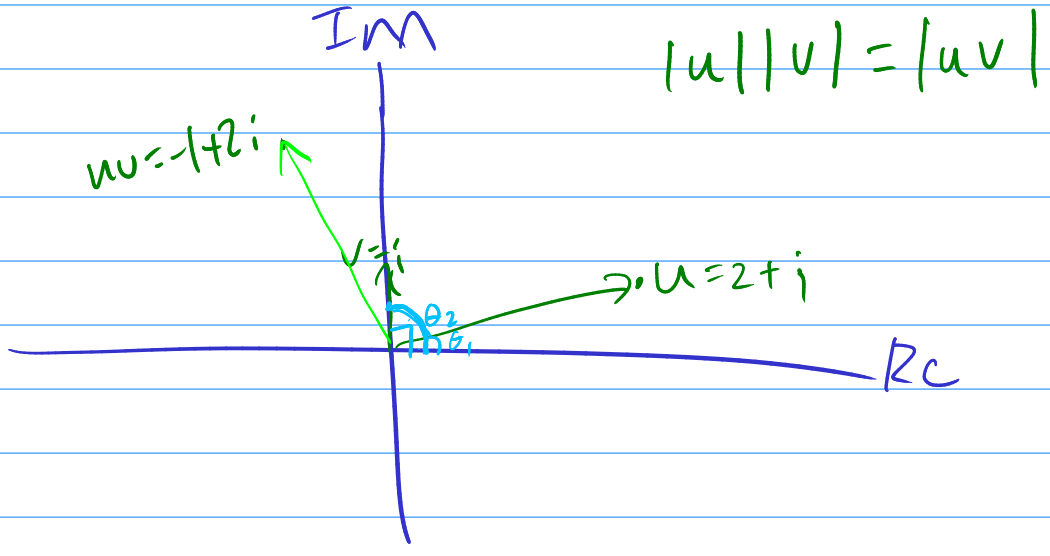
$z\bar{z} = (a+bi)(a-bi) = a^2 + b^2 = |z|^2$

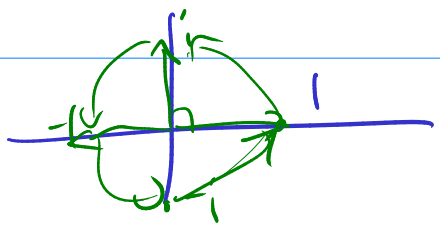
$\frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{4+1} = \frac{2-i}{5}$

$(x+y)(x-y) \quad x^2 - y^2$


complex plane Im
2i
i
z=2+i
|z|
-1
0
1
2
Re
-i
z̄=2-i


Im
u=4+i
Re
u+v=5-4i
v=1-5i


Im
|u||v|=|uv|
uv=-1+2i
v=i
u=2+i
θ2
θ1
Re


i
1
-1
-i