

Divisor Products

What's the product of all the divisors of n ?

Product of the divisors of 24:

$$24 = 2^3 \cdot 3^1$$

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 6 \cdot 8 \cdot 12 \cdot 24 = 24^4$$

$$= n^{\frac{d(n)}{2}}$$

$$n = 60 = 2^2 \cdot 3^1 \cdot 5^1$$

$$d(n) = (2+1)(1+1)(1+1) = 12$$

$$60^{\frac{12}{2}} = 60^6$$

Find the product of the divisors of 450 that are multiples of 3

$$450 = 2^1 \cdot 3^2 \cdot 5^2$$

$$\begin{array}{ccc} 2^0 & & 3^0 \\ 2^1 & 3^1 & 5^1 \\ & 3^2 & 5^2 \end{array}$$

$$2 \cdot 2 \cdot 3 = 12$$

$$150 = 2^1 \cdot 3^1 \cdot 5^2$$

$$\begin{array}{ccc} 2^0 & 3^0 & 5^0 \\ 2^1 & 3^1 & 5^1 \end{array}$$

$$150^{\frac{12}{2}} \cdot 3^{12}$$

$$150^6 \cdot 3^{12}$$

$$50 = 2^1 \cdot 5^2$$

$$50^3$$

Sum of the divisors

$$24 = 2^3 \cdot 3^1$$

1, 2, 3, 4, 6, 8, 12, 24

	2^0	2^1	2^2	2^3
3^0	1	2	4	8
3^1	3	6	12	24

$$(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1)$$

$$1 + 2 + 4 + 8 \quad 1 + 3$$

$$15 \cdot 4 = 60$$

Sum of the divisors of 60,

$$60 = 2^2 \cdot 3^1 \cdot 5^1$$

$$(2^0 + 2^1 + 2^2)(3^0 + 3^1)(5^0 + 5^1)$$

$$\frac{2^3 - 1}{2 - 1} \cdot \frac{3^2 - 1}{3 - 1} \cdot \frac{5^2 - 1}{5 - 1}$$