Factorizations

$x^2+2xy+y^2=(x+y)^2$

$x^2-2xy+y^2=(x-y)^2 = (x-y)(x-y)$

$x^2-xy-xy+y^2$

$x^3+3x^2y+3xy^2+y^3=(x+y)^3 = (x+y)(x+y)(x+y)$

$(x^2+2xy+y)(x+y)$

$x^3-3x^2y+3xy^2-y^3=(x-y)^3 = (x-y)(x-y)(x-y)$

$x^3-x^2y-x^2y-x^2y$

$+xy^2+x^2y+x^2y-y^3$

$$\boxed{\begin{aligned}(x+y)^n &= \underbrace{(x+y)(x+y)(x+y)\cdots(x+y)}_{n}\\ &=\binom{n}{0}x^n+\binom{n}{1}x^{n-1}y+\binom{n}{2}x^{n-2}y^2\\ &\quad+\binom{n}{3}x^{n-3}y^3+\cdots\binom{n}{n}y^n\end{aligned}}$$

Binomial theorem

$\binom{n}{k} = \frac{n\cdot(n-1)(n-2)\cdots(n-k+1)}{k!} = n$ choose k

= # of ways to choose k things out of n things w/o replacement and order doesn't matter

$= \frac{n!}{k!(n-k)!}$

$(x-y)^n = \binom{n}{0}x^n-\binom{n}{1}x^{n-1}y+\binom{n}{2}x^{n-2}y^2-\binom{n}{3}x^{n-3}y^3 \cdots (-1)^n\binom{n}{n}y^n$

$x^2-y^2=(x+y)(x-y)$

$x^2+y^2=(x+yi)(x-yi)$ useless

$x^3-y^3=(x-y)(x^2+xy+y^2)$

$x^3+\cancel{x^2y}+\cancel{xy^2}$

$-\cancel{x^2y}-\cancel{xy^2}-y^3$

$$x^3+y^3=(x+y)(x^2-xy+y^2)$$

$$x^n-y^n=(x-y)(x^{n-1}+x^{n-2}y+x^{n-3}y^2+\dots+y^{n-1})$$

$$x^n+y^n=(x+y)(x^{n-1}-x^{n-2}y+x^{n-3}y^2+\dots+y^{n-1})$$

when n is odd

$$x^4+y^4=(x+y)(x^3-x^2y+xy^2-y^3)$$

$$x^4-x^3y+x^2y^2-xy^3$$
$$-x^3y-x^2y^2+xy^3-y^4=x^4-y^4$$

Ex: $x^3+1000=x^3+10^3=(x+10)(x^2-10x+100)$

Ex: $-a^3b^3-8=-(a^3b^3+2^3)=-(ab+2)(a^2b^2-2ab+4)$

Ex: Find the two prime factors of 7,999,999,999

$8,000,000,000-1$

$$2000^3-1^3=(2000-1)(2000^2+2000+1)$$
$$1999\cdot 4002001$$

Ex: $z^2+\frac{1}{z^2}=14$, find $z^5+\frac{1}{z^5}$

$$\Big(\underbrace{z+\frac{1}{z}}_{4}\Big)^2=z^2+2\cdot z\cdot\frac{1}{z}+\frac{1}{z^2}=\underbrace{z^2+\frac{1}{z^2}}_{14}+2=16$$

$$\left(z^2+\frac{1}{z^2}\right)^2=14^2$$

$$z^4 + \underbrace{2 \cdot z^2 \cdot \frac{1}{z^2}}_{2} + \frac{1}{z^4} = 196$$

$$z^4 + \frac{1}{z^4} = 196$$

$$\left(z + \frac{1}{z}\right)\left(z^4 + \frac{1}{z^4}\right) = z^5 + z^3 + \frac{1}{z^3} + \frac{1}{z^5} = 784$$

$$\left(z + \frac{1}{z}\right)\left(z^2 + \frac{1}{z^2}\right) = \underbrace{z^3 + \frac{1}{z^3}}_{54} + 2 = 4 \cdot 14 = 56$$

$$z^5 + \frac{1}{z^5} = 730$$