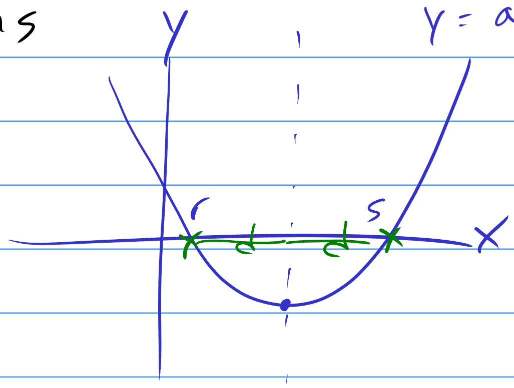
Quadratic Equations

$$ax^2+bx+c=0$$



$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{r+s}{2}=\frac{-\frac{b}{a}}{2}=-\frac{b}{2a}$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}+\frac{c}{a}-\frac{b^2}{4a^2}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0$$

$$r=-\frac{b}{2a}-d$$

$$s=-\frac{b}{2a}+d$$

$$\frac{c}{a}=rs=\left(-\frac{b}{2a}-d\right)\left(-\frac{b}{2a}+d\right)$$

$$\frac{c}{a}=\frac{b^2}{4a^2}-d^2$$

$$d^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

$$d=\frac{\sqrt{b^2-4ac}}{2a}$$

$$x^2+1=0$$

$$x^2=-1$$

$$x=\pm\sqrt{-1}=\pm i$$

$$x^2-4x+7=0$$

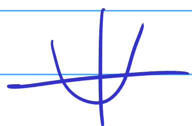
$$\sqrt{12}\cdot\sqrt{-1}$$

$$x=\frac{4\pm\sqrt{16-4\cdot 7}}{2}=2\pm\frac{\sqrt{-12}}{2}=2\pm\frac{\sqrt{12}\,i}{2}$$

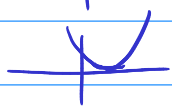
$$=2\pm\sqrt{3}\,i$$

$$\sqrt{b^2-4ac}$$

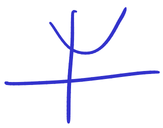
$b^2-4ac>0$



$b^2-4ac=0$



$b^2-4ac<0$



Discriminant

$m+\sqrt{n}$ $\quad m-\sqrt{n}$ rational coefficient

$m+ni$ $\quad m-ni$ real coefficient

Vieta's Formulas for Cubics

$a_3x^3+a_2x^2+a_1x+a_0=0$ roots are $x=r_1, r_2, r_3$

$a_3(x-r_1)(x-r_2)(x-r_3)=0$

$a_3(x^2-(r_1+r_2)x+r_1r_2)(x-r_3)=0$

$a_3(x^3-(r_1+r_2+r_3)x^2+(r_1r_2+r_2r_3+r_1r_3)x-r_1r_2r_3)=0$

$a_2=-a_3(r_1+r_2+r_3) \Rightarrow$
$a_1=a_3(r_1r_2+r_2r_3+r_1r_3) \Rightarrow$
$a_0=a_3(-r_1r_2r_3) \Rightarrow$

$$\boxed{\begin{aligned} r_1+r_2+r_3 &= \frac{-a_2}{a_3} \\ r_1r_2+r_2r_3+r_1r_3 &= \frac{a_1}{a_3} \\ r_1r_2r_3 &= \frac{-a_0}{a_3} \end{aligned}}$$

Ex: $3x^3-4x^2+5x+7=0$ roots are r, s, t

$$r+s+t=\frac{-(-4)}{3}=\frac{4}{3}$$

$$r^2+s^2+t^2=(r+s+t)^2-2rs-2st-2rt=\frac{16}{9}-2\left(\frac{5}{3}\right)=\frac{-14}{9}$$

$r^2+s^2+t^2+2rs+2st+2rt$

$$\frac{1}{r}+\frac{1}{s}+\frac{1}{t}=\frac{st+rt+rs}{rst}=\frac{\frac{5}{3}}{-7/3}=\frac{-5}{7}$$

Vieta's for polynomials

$a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\dots+a_1x+a_0=0$

$a_n(x-r_1)(x-r_2)(x-r_3)\dots(x-r_n)=0$

$a_nx^n-a_n(r_1+r_2+r_3\dots r_n)x^{n-1}+a_n(r_1r_2+r_1r_3+r_1r_4\dots)x^{n-2}-a_n(r_1r_2r_3+r_1r_2r_4\dots)$
$x^{n-3}+\dots(-1)^na_nr_1r_2r_3\dots r_n$

$$\boxed{\begin{aligned} r_1+r_2+r_3+\dots r_n &= -\frac{a_{n-1}}{a_n} \\ r_1r_2+r_1r_3+r_1r_4\dots &= \frac{a_{n-2}}{a_n} \\ &\vdots \\ r_1r_2r_3\dots r_n &= \frac{(-1)^na_0}{a_n} \end{aligned}}$$