The Ninth Grade Math Competition Class
Divisors
Anthony Wang

1. Find the product of the positive divisors of 2400 that are multiples of 6.

2. Find the product of the divisors of 3200 that are perfect squares.

3. A proper divisor of a number is a divisor of the number that is not the number itself. What is the smallest positive integer that is less than the sum of its positive proper divisors?

4. How many positive cubes divide $3! \cdot 5! \cdot 7!$?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. How many of positive divisors of 3200 are not multiples of any perfect square greater than 1?

6. How many positive integers have exactly three proper divisors, each of which is less than 50?

7. Jan is thinking of a positive integer. Her integer has exactly 16 positive divisors, two of which are 12 and 15. What is Jan's number?

8. What is the sum of all positive integers less than 100 that have exactly 12 divisors?

9. Dentoe p_k be the k^{th} prime number. Show that $p_1 p_2 \cdots p_n + 1$ cannot be the perfect square of an integer.

10. Prove that it is impossible for three consecutive squares to sum to another perfect squares.

11. A positive integer n is nice if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \cdots, 2019\}$ are nice?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5 .