

The Ninth Grade Math Competition Class  
Factorials and Palindrome  
Anthony Wang

1. What is the largest 4-digit palindrome that is the sum of 2 different 3-digit palindromes?

[100, 999]

ABBA  
|BB|  
-----

200 1998  
  <sup>1</sup>  <sup>1</sup>  
  5  5  5  
  <sup>6</sup> <sup>6</sup> <sup>6</sup>  
  6  6  6  
1  2  2  1

2. Find the largest  $n$  for which  $12^n$  evenly divides  $20!$ .

$$12^n = 2^{2n} \cdot 3^n$$

$$n \leq 8$$

$$2n \leq 18$$

$$2^1 \quad \frac{20}{2} = 10$$

$$2^2 \quad \frac{10}{2} = 5$$

$$2^3 \quad \frac{5}{2} = 2$$

$$2^4 \quad \frac{2}{2} = 1$$

18

$$3^1 \quad \frac{20}{3} = 6$$

$$3^2 \quad \frac{6}{3} = 2$$

8

3. What is the first year after 2018 that is a palindrome?

A B B A

2 0 0 2

X

2 1 1 2

4. What is the product of the largest 3 digit palindrome and the least 3 digit palindrome?

$$999 \cdot 101 = 100899$$

5. How many 5-digit palindromes are there?

A B C D E

E D C B A

0 1 2 1 0 = 1210

A B C B A

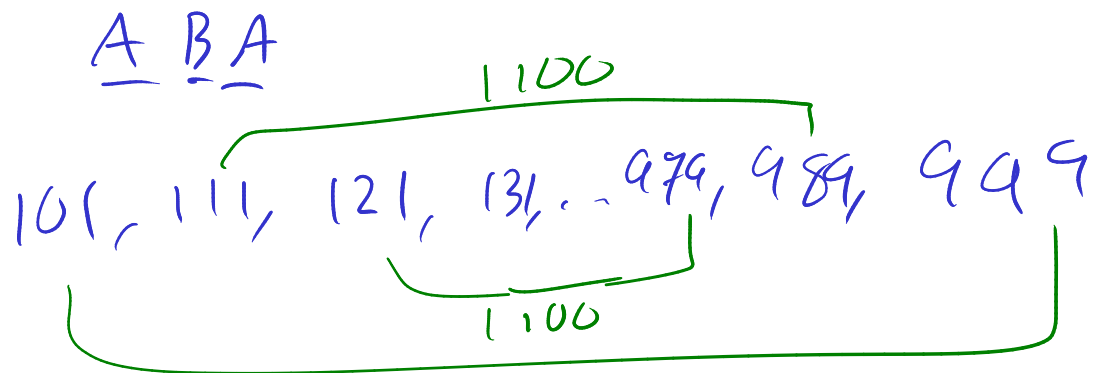
T 0 0 - -

2  
:  
9

1  
:  
9

9 · 10 · 10 = 900

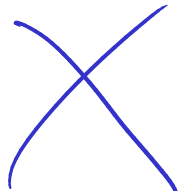
6. Find the sum of all 3-digit plaindromes.

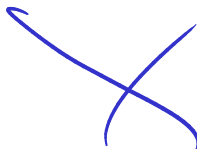



1100

$$1100 \cdot \frac{90}{2} = 49500$$

7. Palindromic primes are numbers that are both palindromic and prime. Find the greatest 3-digit palindromic prime?

$$999 = 9 \cdot 111$$
$$3 \cdot 333$$


$$989 = 23 \cdot 43$$


$$979 = 11 \cdot 89$$


$$969 = 3 \cdot 323$$


$$959 = 7 \cdot 137$$


$$949 = 13 \cdot 73$$


$$939 = 3 \cdot 313$$


$$929 =$$


10001, 10101, 10201, ..., 99799, 99899, 99999

$$S = 110000 \cdot \frac{900}{2} = 110000 \cdot 450 = 49500000$$

8. A five-digit palindrome is a positive integer with respective digits  $abcba$ , where  $a$  is non-zero. Let  $S$  be the sum of all five-digit palindromes. What is the sum of the digits of  $S$ ?

4+9+5+18

9. h There are unique integers  $a_2, a_3, a_4, \dots, a_7$  such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

with  $0 \leq a_i < i$ , for  $i = 2, 3, \dots, 7$ . Find  $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ .

$0 \leq a_7 < 7$

$1+1+0+4+2 = 9$

$$\frac{5}{7} = \frac{a_2}{2} + \frac{a_3}{3 \cdot 2} + \frac{a_4}{4 \cdot 3 \cdot 2} + \frac{a_5}{5 \cdot 4 \cdot 3 \cdot 2} + \frac{a_6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} + \frac{a_7}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$$

$$5 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 a_2 + 7 \cdot 6 \cdot 5 \cdot 4 a_3 + 7 \cdot 6 \cdot 5 a_4 + 7 \cdot 6 a_5 + 7 a_6 + a_7$$

$$5 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 7(6 \cdot 5 \cdot 4 \cdot 3 a_2 + 6 \cdot 5 \cdot 4 a_3 + 6 \cdot 5 a_4 + 6 a_5 + a_6) + a_7$$

$3600 = A = 7B + a_7$

$7 \overline{) 3600} \quad 514 \text{ R } 2$

$$6 \cdot 5 \cdot 4 \cdot 3 a_2 + 6 \cdot 5 \cdot 4 a_3 + 6 \cdot 5 a_4 + 6 a_5 + a_6 = 514$$

$$6(5 \cdot 4 \cdot 3 a_2 + 5 \cdot 4 a_3 + 5 a_4 + a_5) + a_6 = 514$$

85 R 4

$6 \overline{) 514}$

$$5 \cdot 4 \cdot 3 a_2 + 5 \cdot 4 a_3 + 5 a_4 + a_5 = 85$$

$$5(4 \cdot 3 a_2 + 4 a_3 + a_4) + a_5 = 85$$



$$4 \cdot 3a_2 + 4a_3 + a_4 = 17$$
$$4(3a_2 + a_3) + a_4 = 17$$
$$3a_2 + a_3 = 4$$

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