The Ninth Grade Math Competition Class
Factorials and Palindrome
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1. What is the largest 4-digit palindrome that is the sume of 2 different 3-digit palindromes?

[100, 999]

200 1998

ABBA

1 BB 1

5 5 5

6 6 6

1 2 2 1

2. Find the largest n for which 12^n evenely divides $20!$.

$12^n = 2^{2n} \cdot 3^n$

$n \leq 8$

$2n \leq 18$

2^1 $\quad \frac{20}{2} = 10$ $\qquad 3^1$ $\quad \frac{20}{3} = 6$

2^2 $\quad \frac{10}{2} = 5$ $\qquad 3^2$ $\quad \frac{6}{3} = 2$

8

2^3 $\quad \frac{5}{2} = 2$

2^4 $\quad \frac{2}{2} = 1$

18

3. What is the first year after 2018 that is a palindrome?

ABBA

~~2002~~

2112

4. What is the product of the largest 3 digit palindrome and the least 3 digit palindrome?

$999 \cdot 101 = 100899$

5. How many 5-digit palindromes are there?

A B C D E

E D C B A

0 1 2 1 0 = 1210

A B C B A

1 0 0

2 1 1

: 2 :

9 : :

: 9

9

$9 \cdot 10 \cdot 10 = 900$

6. Find the sum of all 3-digit plaindromes.

$\underline{A}\ \underline{B}\ \underline{A}$

1100

101, 111, 121, 131, . . . 979, 989, 999

1100

1100

$$1100 \cdot \frac{90}{2} = 49500$$

7. Palindromic primes are numbers that are both palindromic and prime. Find the greatest 3-digit palindromic prime?

$999 = 9 \cdot 111$

$3 \cdot 333$

$989 = 23 \cdot 43$

$979 = 11 \cdot 89$

$969 = 3 \cdot 323$

$959 = 7 \cdot 137$

$949 = 13 \cdot 73$

$939 = 3 \cdot 313$

$929 =$

$10001,\ 10101,\ 10201,\ \ldots\ 99799, 99899, 99999$

$S = 110000 \cdot \frac{900}{2} = 110000 \cdot 450 = 49500000$

110000

8. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S?

$4+9+5=18$

9. h There are unique integers $a_2, a_3, a_4, \ldots, a_7$ such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

with $0 \le a_i < i$, for $i = 2, 3, \ldots, 7$. Find $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$.

$1+1+1+0+4+2 = 9$

$0 \le a_7 < 7$

$$\frac{5}{7} = \frac{a_2}{2} + \frac{a_3}{3\cdot 2} + \frac{a_4}{4\cdot 3\cdot 2} + \frac{a_5}{5\cdot 4\cdot 3\cdot 2} + \frac{a_6}{6\cdot 5\cdot 4\cdot 3\cdot 2} + \frac{a_7}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}$$

$5\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2 = 7\cdot 6\cdot 5\cdot 4\cdot 3 a_2 + 7\cdot 6\cdot 5\cdot 4 a_3 + 7\cdot 6\cdot 5 a_4 + 7\cdot 6 a_5 + 7 a_6 + a_7$

$5\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2 = 7(6\cdot 5\cdot 4\cdot 3 a_2 + 6\cdot 5\cdot 4 a_3 + 6\cdot 5 a_4 + 6 a_5 + a_6) + a_7$

$3600 = A = 7B + a_7$

$7 \overline{)A}$ B R a_7

$7 \overline{)3600}$ 514 R2

$6\cdot 5\cdot 4\cdot 3 a_2 + 6\cdot 5\cdot 4 a_3 + 6\cdot 5 a_4 + 6 a_5 + a_6 = 514$

$6(5\cdot 4\cdot 3 a_2 + 5\cdot 4 a_3 + 5 a_4 + a_5) + a_6 = 514$

$6 \overline{)514}$ 85 R4

$5\cdot 4\cdot 3 a_2 + 5\cdot 4 a_3 + 5 a_4 + a_5 = 85$

$5(4\cdot 3 a_2 + 4 a_3 + a_4) + a_5 = 85$

$$4 \cdot 3a_2 + 4a_3 + a_4 = 17$$
$$4(3a_2 + a_3) + a_4 = 17$$
$$3a_2 + a_3 = 4$$

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