Factorials

How many ways to arrange 4 books on a shelf?

A B C D

$4 \cdot 3 \cdot 2 \cdot 1 = 24 = 4!$

n books?

$n \cdot (n-1)(n-2) \cdots 2 \cdot 1 = n!$

Find the largest prime divisor of $6! + 5!$

$6! + 5! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$6! = 6 \cdot 5!$

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 (6 + 1)$

$= 7 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Find the largest power of 2 that divides 20!

$20! = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1 \quad 2^1$

$2^2 \quad 2^2 \quad 2^2$

2^3

2^4

10 → 5 → 2 → 1

$2^{1+1+2+1+3+1+2+1=18}$

$2^1 = \frac{20}{2} = 10$

$2^2 = \frac{10}{2} = 5$

$2^3 = \frac{5}{2} = 2$

$2^4 = \frac{2}{2} = 1$

Find the largest power of 3 that divides 42!

3^1 $\frac{42}{3} = 14$

3^2 $\frac{14}{3} = 4$

3^3 $\frac{4}{3} = 1$

(19)

Find the product of divisors of 8!

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 2^7 \cdot 3^2 \cdot 5^1 \cdot 7^1$

$8! \quad \frac{8 \cdot 3 \cdot 2 \cdot 2}{2} = 48$