The Ninth Grade Math Competition Class

Logarithm Challenging Problems

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0. What is the logarithm of $27\sqrt[4]{9}\sqrt[3]{9}$ base 3?

1. Find x if $\log_9(2x-7)=\frac{3}{2}$.

2. Find $\log_{\sqrt{3}}\sqrt[3]{9}$.

3. Solve the equation $\log_{2x} 216 = x$, where x is real.

4. Find base b such that $\log_b 5\sqrt{5}=\frac{5}{2}$.

5. If $\log_2 b - \log_2 a = 3$, then $b^2-a^2=Ma^2$, compute M.

6. If $\frac{\log_b a}{\log_c a}=\frac{19}{99}$, $\frac{b}{c}=c^k$, find the value of k.

7. Let $T=1.8$, compute base b if $\log_b(75T)=2+\log_b 3+\log_b 5$.

8. If $\log_{10} 14 = x$, $\log_{10} 15 = y$, and $\log_{10} 16 = z$, then determne the number of elements in $S = \{\log_{10} 1, \log_{10} 2, \log_{10} 3, \ldots, \log_{10} 100\}$ which can be written in the form $ax+by+cz+d$ for rational numbers a, b, c, d.

9. Two of the vertices of a square are $A\ (\log_{15} 5, 0)$ and $B\ (0, \log_{15} x)$, for $x>1$. The other two vertices lie in the first quadrant. Add the coordinates of all four vertices. The result is 8. Compute x.

10. Given the points $A\ (\log 2, \log 3)$ and $B\ (\log(\log T^2), \log(\log T^3))$, compute the slope of the line $\overleftrightarrow{AB}$.

11. The solutions to the system of equations: (2002 aime 1 #6)

$$\begin{aligned}\log_{225} x + \log_{64} y &= 4\\ \log_x 225 - \log_y 64 &= 1\end{aligned}$$

are (x_1, y_1) and (x_2, y_2). Find $\log_{30}(x_1y_1x_2y_2)$.

12. For integers x and y with $1 < x,\ y \le 100$, compute the number of ordered pairs (x, y) such that $\log_x y + \log_y x^2 = 3$.