Logarithms

Exponents are when we multiply some # by itself repeatedly.

$2^6 = \underbrace{2\cdot2\cdot2\cdot2\cdot2\cdot2}_{6} = 64$

$2^x = 27 \iff x = \log_2 27$

$$\boxed{a^x = b \iff x = \log_a b} \qquad a>0,\ a\neq 1,\ b>0$$

$3^x = 81 \iff x = \log_3 81 = 4$

$\log_4 64 = x \iff 4^x = 64$

$x = 3$

$\log_{2^2} 2^6 = 3$

$$\boxed{\log_{a^m}(a^n) = x \iff (a^m)^x = a^n}$$

$$\boxed{= \frac{n}{m}}$$

Rule 1

$a^{mx} = a^n \qquad mx = n \Rightarrow x = \frac{n}{m}$

$\log_{\frac{1}{2}} \sqrt{2} = \log_{2^{-1}}\left(2^{\frac{1}{2}}\right) = \frac{\frac{1}{2}}{-1} = -\frac{1}{2}$

$\log_8\left(\left(2^{24}\right)^{\frac{3}{4}}\right) = \log_{2^3}\left(2^{18}\right) = \frac{18}{3} = 6$

$\log_{a^m}(b^n) = x \iff (a^m)^x = b^n$

$a^{mx} = b^n$

$\log_a b = \frac{n}{m}x \iff a^{\frac{m}{n}x} = b$

$x = \frac{n}{m}\log_a b$

$$\log_{a^m}(b^n) = \frac{n}{m}\log_a b \quad \text{Rule 2}$$

$$\log_a(bc) = \log_a b + \log_a c \quad \text{Rule 3}$$

x : $a^x = b$; y : $a^y = c$

$\log_a(a^x a^y) = \log_a(a^{x+y}) = x + y$

Ex:

$\log_{10} 2 + \log_{10} 5 = \log_{10}(2 \cdot 5) = \log_{10} 10 = 1$

$$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c \quad \text{Rule 4}$$

$$\log_a b = \frac{1}{\log_b a} \quad \text{Rule 5}$$

$\log_a b = x$

$a^x = b$

$(a^x)^{\frac{1}{x}} = b^{\frac{1}{x}}$

$a = b^{\frac{1}{x}}$

$\log_b a = \frac{1}{x}$

$\log 100 = \log_{10} 100 = 2$

$\ln 100 = \log_e 100$

$\lg 100 = \log_2 100$

$\log_2 27 = \frac{\log 27}{\log 2} = \frac{\ln 27}{\ln 2}$

$\log_b c = \frac{\log_a c}{\log_a b}$ $\log_a c = x \rightarrow a^x = c$

$\log_a b = y \rightarrow a^y = b$

$\log_{a^y} a^x = \frac{x}{y}$

Ex: $\frac{\log_2 36}{\log_2 6} = \log_6 36 = 2$