

Factorizations

$$x^2 + 2xy + y^2 = (x+y)^2$$

$$x^2 - 2xy + y^2 = (x-y)^2 = (x-y)(x-y)$$

$x^2 - xy - xy + y^2$

$$x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3 = (x+y)(x+y)(x+y)$$

$(x^2 + 2xy + y^2)(x+y)$

$$x^3 - 3x^2y + 3xy^2 - y^3 = (x-y)^3 = (x-y)(x-y)(x-y)$$

$x^3 - x^2y - x^2y - x^2y$

$$(x+y)^n = \underbrace{(x+y)(x+y)(x+y) \cdots (x+y)}_n$$

$$= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \dots + \binom{n}{n}y^n$$

Binomial theorem

$$\binom{n}{k} = \frac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k!}$$

$\binom{n}{k} = n$ choose k

= # of ways to choose k things out of n things w/o replacement and order doesn't matter

$$= \frac{n!}{k!(n-k)!}$$

$$(x-y)^n = \binom{n}{0}x^n - \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 - \binom{n}{3}x^{n-3}y^3 + \dots + \binom{n}{n}y^n$$

$$x^2 - y^2 = (x+y)(x-y)$$

$$x^2 + y^2 = (x+yi)(x-yi) \quad \text{useless}$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \dots + y^{n-1})$$

$$x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 + \dots + y^{n-1})$$

when n is odd

$$x^4 + y^4 = (x+y)(x^3 - x^2y + xy^2 - y^3)$$

$$x^4 - x^3y + x^2y^2 - xy^3$$

$$-x^3y - x^2y^2 + xy^3 - y^4 = x^4 - y^4$$

$$\text{Ex: } x^3 + 1000 = x^3 + 10^3 = (x+10)(x^2 - 10x + 100)$$

$$\text{Ex: } -a^3b^3 - 8 = -(a^3b^3 + 2^3) = -(ab+2)(a^2b^2 - 2ab + 4)$$

Ex: Find the two prime factors of
7,999, 999, 999

$$8,000,000,000 - 1$$

$$2000^3 - 1^3 = (2000-1)(2000^2 + 2000 + 1)$$

$$\textcircled{1999} \cdot \textcircled{4002001}$$

$$\text{Ex: } z^2 + \frac{1}{z^2} = 14, \text{ find } z^5 + \frac{1}{z^5}$$

$$\underbrace{\left(z + \frac{1}{z}\right)}_4^2 = z^2 + 2 \cdot z \cdot \frac{1}{z} + \frac{1}{z^2} = z^2 + \frac{1}{z^2} + 2$$

$$14 + 2 = 16$$

$$\left(z^2 + \frac{1}{z^2}\right)^2 = 14^2$$

$$2^4 + 2 \cdot 2^2 \cdot \frac{1}{2^2} + \frac{1}{2^4} = 196$$

$$2^4 + \frac{1}{2^4} = 196$$

$$(2 + \frac{1}{2})(2^4 + \frac{1}{2^4}) = 2^5 + 2^3 + \frac{1}{2^3} + \frac{1}{2^5} = 784$$

$$(2 + \frac{1}{2})(2^2 + \frac{1}{2^2}) = 2^3 + \frac{1}{2^3} + 2 = 4 \cdot 14 = 56$$

$$2^5 + \frac{1}{2^5} = 730$$

Simon's Favorite Factoring Trick

Ex: Find all pairs of positive integers m, n
 $mn + m + n = 76$

$$(m+1)(n+1) = mn + m + n + 1$$

$$mn + m + n + 1 = 77$$

$$(m+1)(n+1) = 77$$

$$7 \quad 11 \quad \Rightarrow m=6, n=10 \quad \checkmark$$

$$1 \quad 77 \quad \Rightarrow \cancel{m=0, n=76}$$

$$11 \quad 7 \quad \Rightarrow m=10, n=6 \quad \checkmark$$

$$mn + am + bn + ab = (m+b)(n+a)$$

Ex: m, n positive integers, solutions to

$$\frac{4}{m} + \frac{2}{n} = 1$$

$$\frac{4n+2m}{mn} = 1$$

$$4n+2m = mn$$

$$8 = mn - 2m - 4n + 8$$

$$8 = (m-4)(n-2)$$

1 8

2 4

4 2

8 1

4 possible

solutions

Ex: Factor $x^2y^2 - 4y^2 - x^2 + 4$

$$(x^2 - 4)(y^2 - 1)$$