The Ninth Grade Math Competition Class
Base Numbers 1
Anthony Wang

1. What is the largest base 10 number that can be expressed as a three-digit base 5 number?

$444_5 = 4 \cdot 5^2 + 4 \cdot 5^1 + 4 \cdot 5^0$

$100 + 20 + 4$

124

$X_{10} = ABC_5$

2. How many natural numbers require 3 digits when written in base 12, but require 4 digits when written in base 9?

$ABC_{12} = WXYZ_9$

$100_{12} = 12^2 = 144$ $1000_9 = 729$

$BBB_{12} = 12^3 - 1 = 1727$ $8888_9 = 6560$

$1727 - 729 + 1 = 999$

3. Given $9^6 = 531441$, how would you represent 531440 in base 9?

$9^6 - 1$

$1000000_9 - 1$

$= 8888888_9$

4. How many integers from 1 to 1992 inclusive have a base-three representation that does not contain the digit 2?

$3\overline{)1992}$ 664 R0

$3\overline{)664}$ 221 R1

$1992 =$

2201210_3

1111111_3

0000001_3

0000000_3

$2^7 - 1 = 127$

5. When written in base 3, a positive integer has two terminal zeros. When written in base 4 or base 5, this same integer has one terminal zero. In how many other positive integral bases greater than 1 must the representation of this integer have at least one terminal zero?

$n_{10} = ABCD\ldots\ |00_3$

$\ldots 3^3\ 3^2\ 3^1\ 3^0$

$n = 3^2 x$

$n_{10} = ABCD\ldots\ |0_4$

$\ldots 4^2\ 4^1\ |\ 4^0$

$n = 4^1 y \qquad n = 5^1 z$

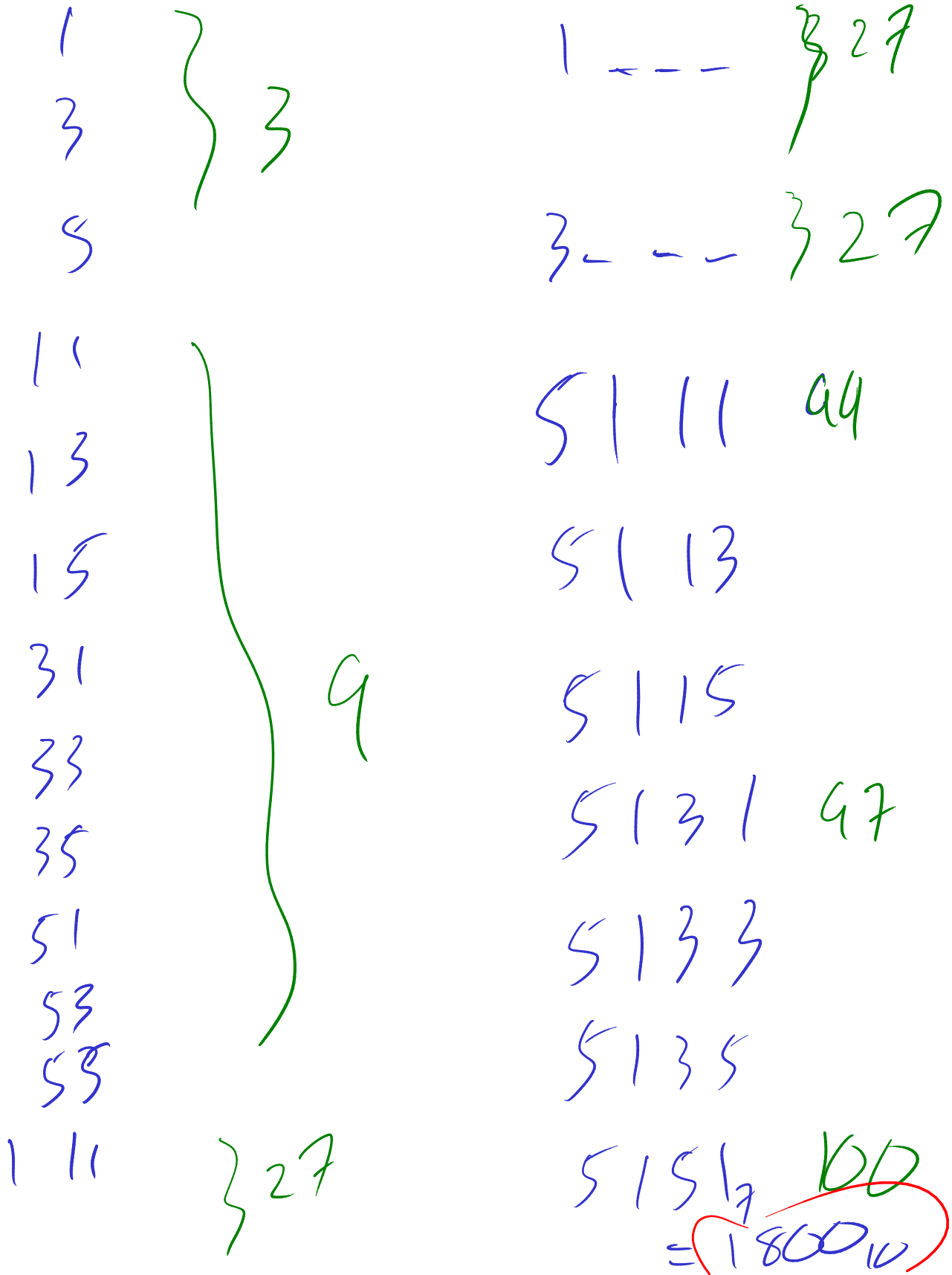
$n = 3^2\ 2^2 5^1 k$

$b|n$, then n has a terminal 0 in base b

$3 \cdot 3 \cdot 2 = 18$

$18 - 3 - 1 = 14$

6. Find the 100^{th} smallest positive integer that can be written using only the digits 1, 3, and 5 in base 7.



7. A number N has three digits when expressed in base 7. When N is expressed in base 9, the digits are reversed. Find the middle digit in either representation of N.

$N = ABC_7 = CBA_9$

$49A + 7B + C = 81C + 9B + A$

$48A - 80C - 2B = 0$

$24A - 40C - B = 0$

$24A - 40C = B$

$8(3A - 5C) = B$

$B = 0$

~~$B = 8$~~