

The Ninth Grade Math Competition Class
Modular Arithmetic
Anthony Wang

a, b

1. The remainders when two natural numbers are divided by 12 are 5 and 9. (a) Find the remainder when their product is divided by 12. (b) Find the remainder when their product is divided by 4.

$$(a) \quad \begin{array}{l} a \\ b \end{array} \begin{array}{l} \text{mod } 12 = 5 \\ \text{mod } 12 = 9 \end{array} \quad \begin{array}{l} a \equiv 5 \pmod{12} \\ b \equiv 9 \pmod{12} \end{array}$$

$$\begin{aligned} ab &\equiv 45 \pmod{12} \\ ab \text{ mod } 12 &= 45 \text{ mod } 12 = 9 \end{aligned}$$

(b)

$$\begin{aligned} ab &= 12q + 9 \\ ab \text{ mod } 4 &= (12q + 9) \text{ mod } 4 \\ &= 1 \end{aligned}$$

2. Is $21^{100} - 12^{100}$ a multiple of 11?

$$(21^{100} - 12^{100}) \pmod{11} = 0 \quad \text{YES}$$
$$(1 - 1) = 0$$

$$21^{100} \pmod{11} = (21 \pmod{11})^{100} \pmod{11}$$
$$= 10^{100} \pmod{11}$$

$$21^{100} \equiv 10^{100} \equiv (-1)^{100} \pmod{11}$$
$$\equiv 1 \pmod{11}$$

$$12^{100} \equiv 1^{100} \equiv 1 \pmod{11}$$

3. Find the remainder when $24^{50} - 15^{50}$ is divided by 13.

4. Find the tens and units digits of 7^{2006} .

5. Find the remainder when $1^2 + 2^2 + 3^2 + \cdots + 99^2$ is divided by 13.

6. Find the remainder when $9^{42} - 5^{42}$ is divided by 7.

7. Find the remainder when 7^{255} is divided by 11.

8. Find the last two digits of 99^{2005} .

9. A natural number n , has a unit digit of A when expressed in base 12. Find the remainder when n^2 is divided by 6.