Exponents

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ times}} = 32$$

$$\underbrace{a}_{\text{base}}^{\overbrace{b}^{\text{exponent}}} = \underbrace{a \cdot a \cdot a \cdots a}_{b \text{ times}}$$

a to the bth power

Ex: $2^3 \cdot 2^4 = 2^7$

$$8 \cdot 16 = 128$$

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{7 \text{ times}} = 2^7$$

Rule 1

$$\boxed{a^m \cdot a^n = a^{m+n}}$$

$$\underbrace{\underbrace{a a \cdots a}_{m \text{ times}} \cdot \underbrace{a \cdot a \cdots a}_{n \text{ times}}}_{m+n \text{ times}}$$

Ex: $\dfrac{3^6}{3^4} = \dfrac{3 \cdot 3 \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}} = 3^2$

$$\boxed{\frac{a^m}{a^n} = \frac{\overbrace{a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{\cdot} \cdots \cancel{a}}^{m \text{ times}}}{\underbrace{\cancel{a} \cdot \cancel{a} \cdot \cancel{\cdot} \cancel{\cdot} \cancel{a}}_{n \text{ times}}} = a^{m-n}}$$

Rule 2

Ex: $3^0 = \underbrace{\quad}_{0 \text{ times}} = ? = 1$

$$3^0 = 3^{1-1} = \frac{3^1}{3^1} = \frac{3}{3} = 1$$

Ex: $3^{-4} = \frac{3^0}{3^4} = \frac{1}{3^4}$

Rule 3

$$\boxed{a^{-m} = a^{0-m} = \frac{a^0}{a^m} = \frac{1}{a^m}}$$

Ex: Solve $3^{2x} = 3^{x-5}$

$2x = x-5 \Rightarrow x = -5$

Ex: $\frac{3^{x^2}}{3^{2x}} = 27$

$$3^{x^2-2x} = 27 = 3^3$$

$x^2-2x = 3$

$x^2-2x-3=0$

$x = 3, -1$

$$(2^3)^5 = \underbrace{2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3}_{5}$$

$$= 2^{3+3+3+3+3} = 2^{3\cdot5} = 2^{15}$$

$$\boxed{(a^m)^n = a^{mn} = (a^n)^m}$$ Rule 4

$$(4^{-3})^{-2} = 4^{(-3)(-2)} = 4^6$$

$$2^6 \cdot 5^6 = \underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{6} \cdot \underbrace{5 \cdot 5 \cdot 5 \cdots 5}_{6}$$

$$= \underbrace{(2\cdot5)(2\cdot5)\cdots(2\cdot5)}_{6}$$

$$= (2\cdot5)^6$$

$$a^m \cdot b^m = (a \cdot b)^m$$

$$\frac{54^5}{27^5} = \frac{54 \cdot 54 \cdot 54 \cdot 54 \cdot 54}{27 \cdot 27 \cdot 27 \cdot 27 \cdot 27} = \left(\frac{54}{27}\right)^5 = 2^5$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Fractional Exponents

$17^{\frac{1}{2}} = x$

$\left(17^{\frac{1}{2}}\right)^2 = x^2$

$17^{\frac{1}{2}\cdot 2} = x^2 = 17$

$\Rightarrow x = \sqrt{17}$

$a^{\frac{1}{n}} = x$

$\left(a^{\frac{1}{n}}\right)^n = x^n$

$a = x^n$

$\Rightarrow \boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

$a^{\frac{m}{n}} = x$

$\left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

$\left(81^{-3}\right)^{\frac{1}{4}} = 81^{-\frac{3}{4}} = \left(3^4\right)^{-\frac{3}{4}} = 3^{-3} = \frac{1}{27}$

$\left(\frac{1}{8}\right)^{\frac{2}{3}} = \left(2^{-3}\right)^{\frac{2}{3}} = 2^{-2} = \frac{1}{4}$

$\frac{4^{\frac{2}{3}}\, 2^{\frac{1}{6}}\, 3^{\frac{3}{2}}}{2^{-\frac{1}{2}}\, 3^{\frac{1}{2}}} = \frac{2^{\frac{4}{3}}\, 2^{\frac{1}{6}}\, 3^{\frac{3}{2}}}{2^{-\frac{1}{2}}\, 3^{\frac{1}{2}}} = 2^{\frac{4}{3}+\frac{1}{6}-\left(-\frac{1}{2}\right)} \cdot 3^{\frac{3}{2}-\frac{1}{2}}$

$= 2^2 \cdot 3^1 = 12$